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CALCULATION OF MAXIMUM DEPTH OF FROST
PENETRATION FOR JOHNS-MANVILLE
WAUKESHA SITE

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The following calculations assume a turf (main-tained grass) surface cover supported by 3 inches of sandy topsoil, covered by a 6-inch cover of snow (the approximate average for the Waukesha area). The sandy topsoil is assumed to have been mixed into the top 3 inches of the cover soil, resulting in a 6-inch thick zone at the surface with thermal properties midway between those of organic silty clay and clean sand.

Inasmuch as the precision of the calculations themselves is approximately $\pm 10\%$, based on the known inconsistency of values for thermal properties, density, and water content, and freezing index, the differences in properties between this surface layer and sand, or between the surface layer and clay, are not taken strictly into account. It is estimated that the resulting error would be in the range 0.5 to 1.0 in., which is less than the overall error in the calculation of maximum frost penetration.

Therefore, in the following calculations, a grass cover is assumed; however, the profile is considered as though it was composed entirely of sand or of clay with thermal and physical properties consistent with those of soils found in the Waukesha vicinity.

For this purpose, maximum penetration depth is considered to be that which would occur in response to temperatures corresponding to the freezing index in the coldest year in 100 years. The probability of

exceedance would be zero in the one-hundred year period, but may be shown to be 0.01 over longer periods of time, which are appropriate to a "permanent" cover.

In the Waukegan area, the freezing index at the $p_e = 0.01$ level is 1950 degF-days, based on National Weather Service records for the most recent 30 years.

a) Maximum penetration depth in SAND.

Coefficients for this calculation, using the Modified Berggren equation are found to be as follows:

$$\lambda = 0.70 ; n_t = 0.65 ; n_s = (0.85)^2 = 0.72 ; F_{0.01} = 1950 ; w_f = 0.30 ; \gamma_{df} = 90.1 ; K_f = 1.50 ; \text{Heave strain} = 10\%.$$

$$\text{and, } : x_{0.01} (\text{ft.}) = \lambda \sqrt{\frac{48 K_f \times n_t \times n_s \times F_{0.01}}{144 \gamma_{df} \times w_f}} = 2.88 \text{ ft} \\ = 34.5 \text{ in.}$$

A complete sequence of calculations is given in the accompanying material.

b) Maximum penetration depth in CLAY.

Similarly, the coefficients in this case are found to be as follows:

$$\lambda = 0.70 ; n_t = 0.65 ; n_s = 0.72 ; F_{0.01} = 1950 ; w_f = \frac{0.50}{0.43} ; \gamma_{df} = \frac{62.9}{72.0} ; K_f = 1.57 ; \text{Heave strain} = 30\%.$$

$$\text{and, } : x_{0.01} (\text{ft.}) = \lambda \sqrt{\frac{48 K_f \times n_t \times n_s \times F_{0.01}}{144 \gamma_{df} \times w_f}} = 2.75 \text{ ft} \\ = 33.0 \text{ in.}$$

As above, the complete sequence of calculations is given in what follows.

a) Maximum penetration depth in SAND.

We assume moderate exposure; $P_e = 0.01$;
Freezing Index $F_{oi} = 1950 \text{ degF-days}$.

The surface is assumed to be grass overlain by snow to an average depth of 6 in.

Dry density (γ_{do}) = 100 pcf ; Water content (w_o) = 0..

Heave Strain^(S) is assumed to be 10% of the frozen soil column; thus, heave strain based on the original soil thickness is 0.11 (11%); frozen dry density (γ_{df}) is $100 / 1.11 = 90.1 \text{ pcf}$.

Assuming a specific gravity^(G) of 2.70, we calculate the fraction of solids in the frozen soil :

$$\lambda_{vf} = \frac{\gamma_{df}}{2.70 \times 62.4} = \frac{90.1}{2.70 \times 62.4} = 0.53$$

Porosity (n_f) is then $(1 - \lambda_{vf}) = 0.47$; the void ratio is

$$e_f = \frac{n_f}{1 - n_f} = \frac{0.47}{0.53} = 0.89$$

It may be shown using dimensional analysis that the following relation is an identity:

$$S_f e_f = G w_f,$$

where S_f is the degree of saturation ($S_f = 0.90$) of the frozen soil, and w_f is the frozen water content. Thus,

$$w_f = \frac{S_f e_f}{G} = \frac{0.90 \times 0.89}{2.70} = 0.30.$$

Using the Modified Berggren equation to calculate depth of penetration in the coldest year in 100 years, we find the coefficients for the Waukegan area to be as follows:

$$= 0.7 \times 4.11 = 2.8871 = \underline{\underline{34.5 \text{ in.}}}$$

$$= 0.7 \sqrt{16.9} = 0.7 \sqrt{\frac{90.1 \times 0.30}{3 \times 1.50 \times 0.45 \times 0.92 \times 1950}} = \frac{f_t \times f_d \times n_s \times F_{0.1}}{144 \times k_f \times n_t \times n_s \times F_{0.1}}$$

The calculation is as follows:

As noted above, freezing index (F_t) is 1950
 $\text{deg}_F\text{-days}$.

Thermal conductivity of the frozen soil
 based on Kestens' relations.
 (K_t) is assumed to be $1.50 \text{ BTU}/\text{deg}_F\text{-hr-ft}$;

Surface coefficients (n_t for turf or grass)
 n_s for 6-in. snow cover) are 0.65
 and $(0.85)^2 = 0.72$ respectively.

(SAND); The climatic coefficient (α) is 0.70.

b) Maximum penetration depth in CLAY (Clayey Silt).

As with SAND, we assume moderate exposure;
 $P_e = 0.01$; Freezing Index $F_{01} = 1950 \text{ degF-days}$.

Also, the surface is assumed to be grass overlain by snow to an average depth of 6 in, as for SAND.

Dry density (γ_{d_0}) = 90 pcf; water content (w_0) = 0.25.

Heave strain (s) is assumed to be 30% for the Waukegan clays (clayey silts) in most years; however, for the coldest years it is generally known that penetration rate will be greater, resulting in somewhat less ice (i.e., segregated ice) being formed during the passage of the freezing front. Therefore, for $P_e = 0.01$, we assume a heave strain of 20%; frozen dry density and water content are calculated accordingly.

Using a specific gravity of 2.65 for the Waukegan clayey soils, and a frozen dry density of $90/1.25 = 72.0 \text{ pcf}$, the fraction of solids in the frozen soil is

$$\lambda_{vf} = \frac{\gamma_{df}}{2.65 \times 62.4} = \frac{72.0}{2.65 \times 62.4} = 0.44$$

Porosity is $(1 - \lambda_{vf}) = 0.56$; void ratio is $e_f = \frac{n_f}{1 - \lambda_f} = 1.27$.

As before with SAND, $s_f e_f = G_w$ is an identity, where

$s_f = 0.90$, the degree of saturation of the frozen soil.

Thus,

$$w_f = s_f e_f / G = \frac{0.90 \times 1.27}{2.65} = 0.43 .$$

The coefficients to the modified Berggren equation, for the coldest year in 100 years in the Waukegan area are as follows:

(CLAY); The climatic coefficient (λ) is 0.70.

Surface coefficients for grass and 6-in. snow are $n_t = 0.65$; $n_s = (0.85)^2 = 0.72$.

Thermal conductivity of the frozen soil (k_f) is 1.57 Btu/degF-hr-ft².

Freezing index (F_{01}) is 1950 degF-days.

The calculation of max. penetration depth is then:

$$\begin{aligned} x_{01} (\text{ft}) &= \lambda \sqrt{\frac{48 \times k_f \times n_t \times n_s \times F_{01}}{144 \times y_{df} \times w_f}} \\ &= 0.7 \sqrt{\frac{\frac{1}{3} \times 1.57 \times 0.65 \times (0.85) \times 1950}{72.0 \times 0.43}} \\ &= 0.7 \sqrt{15.4} \\ &= 0.7 \times 3.93 = 2.75 \text{ ft} = \underline{\underline{33.0 \text{ in.}}} \end{aligned}$$

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(signed)

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